

1.

$$\int_{-1}^2 x^3 dx = \left[\frac{x^4}{4} \right]_{-1}^2 = \frac{16}{4} - \frac{1}{4} = 4 + \frac{1}{4} = \frac{15}{4}$$

2.

$$\int_0^{\frac{\pi}{2}} \cos x dx = [\sin x]_0^{\frac{\pi}{2}} = 1 - 0 = 1$$

3.

$$\int_a^b \frac{dx}{x} = [\ln |x|]_a^b = \ln |b| - \ln |a| = \ln \left| \frac{b}{a} \right|$$

4.

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}} = [\arcsin x]_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$$

5.

$$\begin{aligned} \int_0^{\ln 2} x e^{-x} dx &= \left| \begin{array}{ll} u = x & u' = 1 \\ v' = e^{-x} & v = -e^{-x} \end{array} \right| = [-x e^{-x}]_0^{\ln 2} - \int_0^{\ln 2} (-e^{-x}) dx \\ &= [-x e^{-x}]_0^{\ln 2} + [-e^{-x}]_0^{\ln 2} = -\frac{1}{2} \ln 2 - 0 - \frac{1}{2} + 1 = \frac{1}{2}(1 - \ln 2) \end{aligned}$$

6.

$$\begin{aligned} \int_0^{\pi} x \sin x dx &= \left| \begin{array}{ll} u = x & u' = 1 \\ v' = \sin x & v = -\cos x \end{array} \right| \\ &= [-\cos x \cdot x]_0^{\pi} - \int_0^{\pi} -\cos x dx = [-\cos x \cdot x]_0^{\pi} + [\sin x]_0^{\pi} = \pi + 0 = \pi \end{aligned}$$

7.

$$\begin{aligned} \int_0^1 x(2-x^2)^5 dx &= \left| \begin{array}{lll} 2-x^2 & = & t \quad x=1 \Rightarrow t=1 \\ -2x dx & = & dt \quad x=0 \Rightarrow t=2 \end{array} \right| = -\frac{1}{2} \int_2^1 t^5 dt \\ &= \frac{1}{2} \int_1^2 t^5 dt = \frac{1}{2} \left[\frac{t^6}{6} \right]_1^2 = \frac{64}{12} - \frac{1}{12} = \frac{63}{12} \end{aligned}$$

8.

$$\int_1^{e^8} \frac{dx}{x\sqrt{\ln x + 1}} = \left| \begin{array}{lll} \ln x + 1 & = & t \quad x = e^8 \Rightarrow t = 8 + 1 \\ \frac{1}{x} dx & = & dt \quad x = 1 \Rightarrow t = 0 + 1 \end{array} \right| = \int_1^9 \frac{dt}{\sqrt{t}} = [2\sqrt{t}]_1^9 = 6 - 2 = 4$$

9. Vypočítejte obsah plochy ohraničené křivkami

$$0 = (x+2)(x-1)$$

$$y = 2 - x, y = x^2$$

$$\int_{-2}^1 (2 - x - x^2) dx = \left[2x - \frac{x^2}{2} - \frac{x^3}{3} \right]_{-2}^1$$

$$2 - x = x^2 \rightarrow$$

$$0 = x^2 + x - 2 \rightarrow$$

$$= 2 - \frac{1}{2} - \frac{1}{3} - \left(-4 - \frac{4}{2} - \frac{-8}{3} \right) = 5 - \frac{1}{2} = \frac{9}{2}$$

10.

$$y = 2^x, y = 2, x = 0$$

$$\int_0^1 (2 - 2^x) dx = \left[2x - \frac{2^x}{\ln 2} \right]_0^1$$

$$2^x = 2 \Rightarrow x = 1$$

$$= 2 - \frac{2}{\ln 2} - \left(0 - \frac{1}{\ln 2} \right) = 2 - \frac{1}{\ln 2}$$

11.

$$y = x^2, y = \frac{1}{4}x^2, y = 4$$

$$4 = x^2 \Rightarrow x = \pm 2$$

$$4 = \frac{1}{4}x^2 \Rightarrow x = \pm 4$$

$$x^2 = \frac{1}{4}x^2 \Rightarrow x = 0$$

$$S = 2S_1$$

$$S_1 = \int_0^2 \left(x^2 - \frac{x^2}{4} \right) dx + \int_2^4 \left(4 - \frac{x^2}{4} \right) dx = \left[\frac{x^3}{3} - \frac{x^3}{12} \right]_0^2 + \left[4x - \frac{x^3}{12} \right]_2^4 = \frac{8}{3} - \frac{8}{12} + 16 - \frac{16}{3} - 8 + \frac{8}{12} = \frac{16}{3}$$

$$S = \frac{32}{3}$$

12. Vypočítejte objem komolého kužele s poloměrem podstav r_1, r_2 a výškou v .

Lichoběžník $[0, r_2], [v, r_1], [0, 0], [v, 0]$ necháme rotovat kolem osy x .

$$y = kx + q, \quad k = \frac{r_1 - r_2}{v}, \quad q = r_2$$

$$y = \frac{r_1 - r_2}{v}x + r_2$$

$$V = \pi \int_a^b f^2(x) dx = \pi \int_0^v \left(\frac{r_1 - r_2}{v}x + r_2 \right)^2 dx = \pi \int_0^v \left(\frac{(r_1 - r_2)^2}{v^2}x^2 + 2\frac{(r_1 - r_2)r_2}{v}x + r_2^2 \right) dx$$

$$= \pi \left[\frac{(r_1 - r_2)^2}{3v^2}x^3 + \frac{(r_1 - r_2)r_2}{v}x^2 + r_2^2x \right]_0^v = \frac{1}{3}\pi v(r_1^2 + r_1r_2 + r_2^2)$$

13. Vypočítejte délku obkrouky křivky

$$y = \ln x, x_1 = \sqrt{3}, x_2 = \sqrt{8}.$$

$$l = \int_{x_1}^{x_2} \sqrt{1 + (f')^2} dx = \int_{\sqrt{3}}^{\sqrt{8}} \sqrt{1 + \frac{1}{x^2}} dx = \int_{\sqrt{3}}^{\sqrt{8}} \frac{\sqrt{1 + x^2}x}{x^2} dx$$

$$= \left| \begin{array}{lll} 1 + x^2 & = & t^2 \\ 2x dx & = & 2t dt \end{array} \right| \begin{array}{l} x = \sqrt{8} \Rightarrow t = 3 \\ x = \sqrt{3} \Rightarrow t = 2 \end{array} = \int_2^3 \frac{t^2}{t^2 - 1} dt = \int_2^3 \left(1 + \frac{1}{t^2 - 1} \right) dt$$

$$= \int_2^3 dt + \frac{1}{2} \int_2^3 \left(\frac{1}{t-1} - \frac{1}{t+1} \right) dt = [t]_2^3 + \frac{1}{2} [\ln |t-1| - \ln |t+1|]_2^3 = 1 + \frac{1}{2} \ln \left(\frac{3}{2} \right)$$