

Neural Network Based Representation of Time-Delay System Models

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Abstract:

Time delay models have proved useful in many application directions, from distributed parameter systems, systems with transportation delays, supply chain management, population dynamics to networked and multi-agent systems. A mathematical description of time delay systems (TDS) has been introduced by a functional generalization of ordinary differential equations. Conventional modeling and identification approaches usually assume a priori knowledge of a model structure (order, number of time delays, nonlinear parts, etc.). However, in many cases, no a priori information is provided, and making blind assumptions might be a dull way. Therefore, more complex models are often used. Neural networks (NNs) can be used to learn dynamic systems and simulate their dynamics efficiently. For instance, neural ordinary differential equations (NODEs) express the right-hand side of the differential equation, while the evolution of the network for a given period provides the map from the initial condition to the terminal state. Time delays can also be incorporated into NNs. Delays can either be fixed or considered within the trained parameter set. A suitable approximation can transform a delay differential equation to an ordinary differential equation, enabling the direct use of a NODE.

A doctoral student in their dissertation thesis should use NODEs and/or NDDEs to represent continuous-time or discretized TDS. Weights, biases, and delays within the NN can be estimated via several parameter optimization methods, which can be benchmarked. Besides, the student may consider various discretization techniques of continuous time-delay models (e.g., semi-discretization, full-discretization, and partial spectral discretization methods). In addition, as the NN representation of the so-called universal differential equations has proven efficient for large-scale systems, the student may attempt to extend this approach to systems with delays.

Literature:

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